Arithmetic

Geometric

$$\left[t_n = t_1 + (n-1)d \quad S_n = \frac{n}{2}(t_1 + t_n) \quad S_n = \frac{n}{2}(2t_1 + d(n-1))\right] \quad \left[t_n = t_1 r^{n-1} \quad S_n = \frac{t_1(1-r^n)}{1-r} \quad S_n = \frac{rt_n - t_1}{r-1} \quad S = \frac{t_1}{1-r}\right]$$

Review [Practice Test]

A. Arithmetic Sequences:

1. Find the common difference (d) for the following arithmetic sequence. *3, 13, 23, 33, 43, ...* $d=10$
2. Find the 6^{th} term of the following arithmetic sequence. *\$250.⁰⁰, \$525.⁰⁰, \$800.⁰⁰*, 1075, 1350, $\underline{1625}$ \$ t_6 $d = \$275$
3. Insert three numbers between 15 and -6, so the five numbers form an arithmetic sequence. 15, 9.75, 4.50, -0.75, -6 $\quad 15 + 4d = -6$ $4d = -21$ $d = -5.25$
4. Given the following arithmetic sequence, 4, 9, 1 4, ..., *a) state the common difference (d):* $\underline{5}$ *b) determine the general term of the sequence:* $t_n = 4 + 5(n-1)$ $t_n = 4 + 5n - 5$ $\underline{t_n = 5n - 1}$ *c) find the 25^{th} term of the sequence:* $t_{25} = 5(25) - 1$

$\underline{t_{25} = 124}$

Arithmetic | Geometric

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5. *Given that the general term of a sequence is* $t_n = 13 - 17n$, *find the first four terms of the sequence.*

$t_1 = 13 - 17(1) = -4$

$t_2 = 13 - 17(2) = -21$

$t_3 = 13 - 17(3) = -38$

$t_4 = 13 - 17(4) = -55$

6. *Given the following arithmetic sequence, 10, 2, −6 ...find* t_{122}.

$t_1 = 10$

$d = -8$

$n = 122$

$t_{122} = 10 + (121)(-8)$

$t_{122} = -958$

7. *In the following arithmetic sequence, 14, 28, 42, 56, ..., 1470, which term is 1470?*

$t_n = 1470$

$t_1 = 14$

$d = 14$

$1470 = 14 + (n-1)14$

$1470 = \cancel{14} + 14n - \cancel{14}$

$105 = n$

Arithmetic

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Geometric

$$\left[t_n = t_1 r^{n-1} \quad S_n = \frac{t_1(1-r^n)}{1-r} \quad S_n = \frac{rt_n - t_1}{r-1} \quad S = \frac{t_1}{1-r}\right]$$

B. Arithmetic Series:

1. Given the following arithmetic series, 3 + 13 + 23 + 33,..., find the sum of the first 40 terms. $t_1 = 3,\ d = 10,\ n = 40$ $t_{40} = 3 + (39)(10)$ $\therefore t_{40} = 393$ $S_{40} = \frac{40}{2}(3+393)$ $S_{40} = 20(396)$ $\therefore S_{40} = 7920$
2. Find the sum of the following arithmetic series, 5 + 3 + 1 + ... + (-53). $-53 = 5 + (n-1)(-2)$ $-53 = 5 - 2n + 2$ $-60 = -2n$ $30 = n$ $S_{30} = \frac{30}{2}(5 + (-53))$ $S_{30} = 15(-48)$ $S_{30} = -720$
3. Find the sum of the following arithmetic series, 2 + 10 + 18 + ... + 274. $274 = 2 + (n-1)8$ $274 = 2 + 8n - 8$ $280 = 8n$ $35 = n$ $S_{35} = \frac{35}{2}(2+274)$ $S_{35} = 17.5(276)$ $S_{35} = 4830$
4. Given that the 3rd term of a series is 7 and that the 7th term is 31, Find the sum of the first 50 terms in the series. $-5,\ 1,\ 7,\ \_,\ \_,\ \_,\ 31$ $7 + 4d = 31$ $4d = 24$ $d = 6$ $\therefore t_1 = -5$ $S_{50} = \frac{50}{2}(2(-5) + 6(50-1))$ $S_{50} = 25(-10 + 294)$ $\therefore S_{50} = 7100$

Arithmetic

$$\left[t_n = t_1 + (n-1)d \quad S_n = \frac{n}{2}(t_1 + t_n) \quad S_n = \frac{n}{2}(2t_1 + d(n-1))\right]$$

Geometric

$$\left[t_n = t_1 r^{n-1} \quad S_n = \frac{t_1(1-r^n)}{1-r} \quad S_n = \frac{rt_n - t_1}{r-1} \quad S = \frac{t_1}{1-r}\right]$$

A little tougher…

1. *In an arithmetic sequence* $t_5 + t_{13} = 198$ *and the common difference [d] is 13. Determine the first term.*

$t_5 = t_1 + 4(13)$ $\quad t_{13} = t_1 + 12(13)$

$t_5 = t_1 + 52$ $\quad t_{13} = t_1 + 156$

$\therefore 2t_1 + 52 + 156 = 198$

$2t_1 = 198 - 208$

$2t_1 = -10$

$\underline{\underline{t_1 = -5}}$

2. *If* $S_4 = 64$ *and* $S_5 = 90$ *determine the first term of the sequence if the common difference is 4.*

$S_5 - S_4 = t_5$

$90 - 64 = t_5$

$\underline{\underline{26 = t_5}}$

$S_5 = (t_5 + t_1)\frac{n}{2}$

$\frac{2}{5} \cdot 90 = (26 + t_1)\frac{5}{2} \cdot \frac{2}{5}$

$36 = 26 + t_1$

$\underline{10 = t_1}$

3. *Determine the common difference in the following arithmetic sequence.*

$t_1 + 2d,\ 3t_1,\ 5t_1 - 2d\ ,7t_1 - 4d$

$3t_1 - (t_1 + 2d)$

$= \underline{\underline{2t_1 - 2d}}$

Arithmetic

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Geometric

$\left[t_n = t_1 r^{n-1} \quad S_n = \frac{t_1(1-r^n)}{1-r} \quad S_n = \frac{rt_n - t_1}{r-1} \quad S = \frac{t_1}{1-r}\right]$

C. Geometric Sequences:

1. What is the 6th term in the following geometric sequence?

-3, -15, -75, ...

$r = 5$

$t_6 = -3(5)^5$

$t_6 = -9375$

2. Insert two numbers between - 2 and 128 so the four numbers form a Geometric Sequence.

$-2, \underline{8}, \underline{-32}, 128$

$-2r^3 = 128$

$r^3 = -64$

$r = -4$

3. Find the 75th term in the following geometric sequence: 4, -4/3, 4/9, …

$r = -\frac{4}{3} \cdot \frac{1}{4} = -\frac{1}{3}$

$t_{75} = 4\left(-\frac{1}{3}\right)^{74}$

$t_{75} = -1.97 \times 10^{-35}$

4. A flea market is conducting a Dutch auction on an item that sells for 320 dollars. It will take off 5 % off the cost of the item each day the item has not sold. What will be the cost on the 7th day? (Important: The item will be worth 95% of the previous day's cost.) $\therefore r = 0.95$

$n = 7$ terms (7 days starting @ day 1)

$t_7 = 320(0.95)^6$

$t_7 = \$235.23$

$100\% + 7.3\% = 107.3\%$

5. A business person bought an investment property for \$1 530 000.00 . If it appears by 7.3 % $r = 1.073$ each year, then what should the property be worth in 4 years? $n = 5$ (starting @ 0 years)

$t_5 = 1\,530\,000(1.073)^4$

$t_5 = \$2\,028\,104.45$

6. In a geometric sequence $t_3 = 45$ and $t_6 = 1\,215$. Determine the first term.

$45r^3 = 1215$

$r^3 = 27$

$r = 3$

$\therefore t_1 = \frac{45}{9}$

$t_1 = 5$

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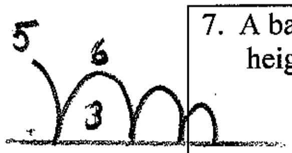


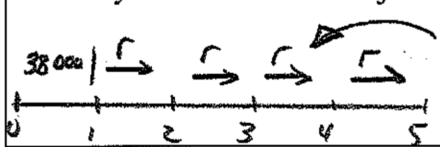
7. A ball is dropped from a height of 5 m. After each bounce, it rises to 60% of its previous height. What is the total vertical distance the ball travels before it comes to rest? $r = 0.60$

S_∞

$\therefore S_\infty = \frac{6}{1-0.60} + 5 = \underline{20m}$

$r = 102\%$ or 1.02%

8. Ben has a starting salary of $38 000 . He receives a 2% increase each year on the previous years salary. He remains on the job for 5 years. What are his total earnings?



$n = 5$

$S_5 = \frac{38000(1-1.02^5)}{1-1.02} = \$197\,753.53$

9. Determine the sum of the following infinite geometric series, if possible.

a) $8+4+2+1+\ldots$ $r = \frac{1}{2}$ "Convergent"

$S_\infty = \frac{8}{1-\frac{1}{2}}$

$\underline{S_\infty = 16}$

b) $1-1.5+2.25-\ldots$ $r = \frac{-1.5}{1}$

$\therefore$ No finite sum

"Divergent"

10. An oil well pumps 5 000 000 barrels of oil in the first month. Each month after the first month, production **DROPS** by 7%. ***(reminder…choose your ratio carefully!)*** $r = 93\%$ or 0.93

a) How much oil will be pumped in the 15^{th} month

$t_{15} = 5\,000\,000(0.93)^{14} = \underline{1\,810\,219.688}$ barrels

b) How many barrels are pumped in total in the first 15 months?

$S_{15} = \frac{5\,000\,000(1-0.93^{15})}{1-0.93} = \underline{47\,378\,509.85}$

c) How many barrels are pumped before the well runs dry?

$S_\infty = \frac{5\,000\,000}{1-0.93} = \underline{71\,428\,571.43}$ barrels