WCP P. Ma 11 **Analysis of Equations Review Worksheet**

Topics covered:

1. Quadratic Formula: $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2. Discriminant:
   $b^2 - 4ac > 0$ two real different roots
   $b^2 - 4ac = 0$ 1 real root (2 equal roots)
   $b^2 - 4ac < 0$ no real roots

3. Remainder Theorem: P(b) = R i.e, $(x^2 - 4x + 2) \div (x - 2)$, then P(2) = 4 – 8 + 2 = -2; therefore, -2 is the Remainder.

4. Factor Theorem: If P(b) = 0, then (x – b) is a factor of P(x). This is when you would use Synthetic division and the integral/rational factor theorem.

5. Polynomial Inequalities. > 0 look above the x–axis for answers and < 0 look below the x-axis.

Review Exercises:

1. Solve the following: (no decimal answers)

a. $3x^2 - 4x - 6 = 0$

$x = \dfrac{4 \pm \sqrt{16+72}}{6}$

$x = \dfrac{4 \pm \sqrt{88}}{6}$

$x = \dfrac{4 \pm 2\sqrt{22}}{6}$

$x = (2 \pm \sqrt{22})/3$

b. $x(x+1) = 15$

$x^2 + x - 15 = 0$

$x = \dfrac{-1 \pm \sqrt{1+60}}{2}$

$x = \dfrac{-1 \pm \sqrt{61}}{2}$

c. $-x^2 - 8x + 2 = 0$

$x^2 + 8x - 2 = 0$

$x = \dfrac{-8 \pm \sqrt{64+8}}{2}$

$x = \dfrac{-8 \pm \sqrt{72}}{2}$

$x = \dfrac{-8 \pm 6\sqrt{2}}{2}$

$x = -4 \pm 3\sqrt{2}$

2. Solve the following to two decimal places. $4x^2 - 5x - 6 = 0$

$x = -0.75$ ¢ 2.00

Graphing calculator or Quad formula completely solved.

3. Indicate the nature of the roots:

a) $2x^2 - 3x + 1 = 0$

$b^2 - 4ac$
$= 9 - 8$
$= 1\ [>0]$
$\therefore$ 2 different roots

b) $7x^2 - 10x + 9 = 0$

$= 100 - 252$
$= -152\ [<0]$
$\therefore$ No real roots

c) $-2x^2 + 2x + 3 = 0$

$= 4 + 24$
$= 28\ [>0]$
$\therefore$ 2 different roots

4. Determine the remainder for the following:

a) $(3x^3 - 2x^2 + 7x - 6) \div (x + 1)$

$f(-1) = 3(-1)^3 - 2(-1)^2 + 7(-1) - 6$
$f(-1) = -3 - 2 - 7 - 6 = \underline{\underline{-18}}$

b) $(x^4 + x^3 - x^2 - 2x - 3) \div (x - 2)$

$f(2) = 16 + 8 - 4 - 4 - 3$
$f(2) = \underline{\underline{13}}$

c) $(x^4 + x^3 - x^2 - 2x - 3) \div (2x + 5)$

$f(\frac{-5}{2}) = (\frac{-5}{2})^4 + (\frac{-5}{2})^3 - (\frac{-5}{2})^2 - 2(\frac{-5}{2}) - 3$
$f(-5/2) = 19.1875$ or $307/16$

5. For $P(x) = 2x^2 - 3x - 2$, find the value of P(-2).

$P(-2) = 2(-2)^2 - 3(-2) - 2$
$P(-2) = \underline{\underline{12}}$

6. State whether the binomial is a factor:

a) $(x^3 + 2x^2 + 2x + 1) \div (x + 1)$

$f(-1) = -1 + 2 - 2 + 1$
$f(-1) = 0$
$\therefore$ $(x+1)$ is a factor

b) $(6x^4 + 2x^3 - 3x^2 - 1) \div (x + 2)$

$f(-2) = 6(-2)^4 + 2(-2)^3 - 3(-2)^2 - 1$
$f(-2) = 96 - 16 - 12 - 1$
$f(-2) \neq 0$
$\therefore$ NOT a factor

7. Solve by factoring:

a) $2x^3 + 3x^2 - 11x - 6 = 0$

$f(-3) = 2(-3)^3 + 3(-3)^2 - 11(-3) - 6$
$= -54 + 27 + 33 - 6$
$= 0$ $\quad\therefore (x+3)$ is a factor

-3	2	3	-11	-6
	↓	-6	9	6
	2	-3	-2	0

$\therefore (x+3)(2x^2 - 3x - 2) = 0$
$\therefore (x+3)(2x+1)(x-2)$
$\therefore x = -3, -\frac{1}{2}, 2$

b) $x^3 - 5x^2 + x + 10 = 0$

$f(2) = 8 - 20 + 2 + 10$
$f(2) = 0$
$\therefore (x-2)$ is a factor

2	1	-5	1	10
	↓	2	-6	-10
	1	-3	-5	0

Quad Formula

$\therefore (x-2)(x^2 - 3x - 5) = 0$
$\therefore x = 2, (3 \pm \sqrt{29})/2$

* Graph with TI 83 to find a "Zero" *

c) $x^3 + 2x^2 - 9x - 18 = 0$

$f(3) = 27 + 18 - 27 - 18$

$f(3) = 0 \therefore (x-3)$ is a factor

$$\begin{array}{r|rrr|r} 3 & 1 & 2 & -9 & -18 \\ & \downarrow & 3 & 15 & 18 \\ \hline & 1 & 5 & 6 & 0 \end{array}$$

$(x^2 + 5x + 6)$

$\therefore (x-3)(x+3)(x+2) = 0$

$\therefore x = -2, \pm 3$

d) $x^4 + 4x^3 - 7x^2 - 34x - 24 = 0$

$f(3) = 81 + 108 - 63 - 102 - 24$

$f(3) = 0$

$\therefore (x-3)$ is a factor

$$\begin{array}{r|rrrr|r} 3 & 1 & 4 & -7 & -34 & -24 \\ & \downarrow & 3 & 21 & 42 & 24 \\ \hline & 1 & 7 & 14 & 8 & 0 \end{array}$$

$\therefore (x+3)(x^3 + 7x^2 + 14x + 8) = 0$

$f(-4) = 0 \therefore (x+4)$ is also a factor (TI 83)

$$\begin{array}{r|rrr|r} -4 & 1 & 7 & 14 & 8 \\ & \downarrow & -4 & -12 & -8 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$\therefore (x^2 + 3x + 2)(x-3)(x+4)$

$\therefore (x+2)(x+1)(x-3)(x+4)$

$\therefore x = -2, -1, 3, -4$

Yikes! So many steps!

8. Determine the value(s) of k such that . . .

a) $8x^2 + kx + 2 = 0$, such that there will be two different roots.

$k^2 - 64 > 0$ [$k^2 - 64 = 0 \Rightarrow$ equal]

$\therefore k < -8$ & $k > 8$

$-8 \qquad 8$

b) $x^2 - 8x + k = 0$, such that there will be one real root.

$64 - 4k = 0$

$64 = 4k$

$16 = k$

c) $2kx^2 + 3x + 1 = 0$, such that there will be no real roots.

$9 - 8k < 0$

$9 < 8k$

$\frac{9}{8} < k$

$\therefore k > \frac{9}{8}$

9. For $(4x^3 - 4x^2 + 2x + k) \div (x + 5)$, find the value of k if the remainder is -613.

$f(-5) = 4(-5)^3 - 4(-5)^2 + 2(-5) + k$

$= -500 - 100 - 10 + k$

$= -610 + k$ [Remainder]

$\therefore k - 610 = -613$

$k = -3$

10. The divisions of $(2x^3+4x^2-kx+5)\div(x+3)$ and $(6x^3-3x^2+2x+7)\div(2x-1)$ have the same remainder. Find the value of k.

$f(-3) = -54+36+3k+5$

$f(-3) = 3k-13$

$g(\tfrac{1}{2}) = 6(\tfrac{1}{8}) - 3(\tfrac{1}{4}) + 1 + 7$

$g(\tfrac{1}{2}) = \tfrac{6}{8} - \tfrac{3}{4} + 8 = 8$

$\therefore\ 3k-13 = 8$

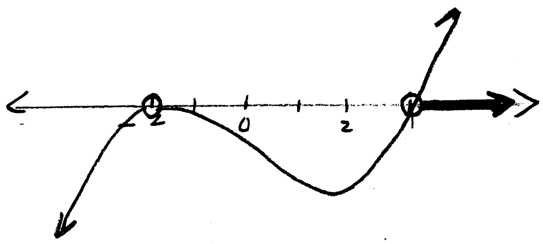
$3k = 21$

$\underline{k=7}$

"Remainders are equal"

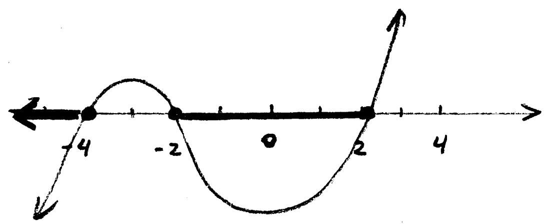
11. Solve the following inequalities:

a) $(x-3)(x+2)^2 > 0$



$\underline{x > 3}$

b) $(x-2)(x+2)(x+4) \le 0$

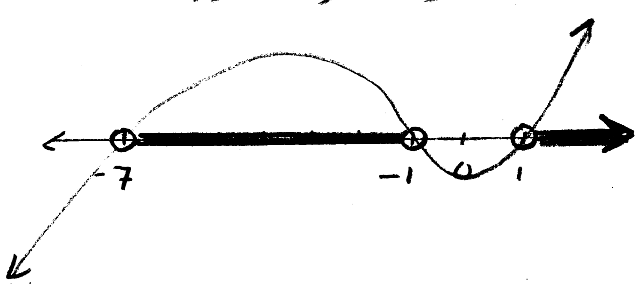


$\underline{x \le -4}$ & $\underline{-2 \le x \le 2}$

13. Solve the following inequalities. Use the graphing calculator for checking your work.

a) $x^3+7x^2-x-7>0$

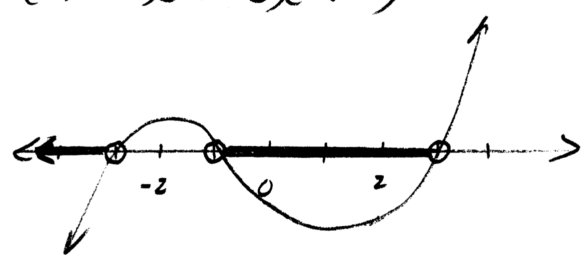
$(x+7)(x+1)(x-1) > 0$



$\underline{x > 1}$ & $\underline{-7 < x < -1}$

b) $x^3+x^2-9x-9<0$

$(x+3)(x-3)(x+1) < 0$



$\underline{x < -3}$ & $\underline{-1 < x < 3}$

* Graph with TI 83 & find the zeros *